

WHAT MAY BE LEARNED IN EXPERIMENTAL STUDY OF RADIATIVE K_{l3} DECAY?

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The possibility of the experimental study of the Chiral Perturbative Theory contributions to the Structure Dependent radiation in $K^+ \rightarrow l^+ \nu \pi^0 \gamma$ decay is discussed. It is shown that experiment may be sensitive only to the contributions due to the counterterm L_9 and chiral anomaly. However, simultaneous independent study of these contributions seems to be unlikely.

1. INTRODUCTION

The radiative K_{l3} decays

$$K^+(p) \rightarrow \pi^0(p') l^+(p_l) \nu_l(p_\nu) \gamma(q) \quad [K_{l3\gamma}^+],$$

$$K^0(p) \rightarrow \pi^-(p') l^+(p_l) \nu_l(p_\nu) \gamma(q) \quad [K_{l3\gamma}^0],$$

have been recently considered in the framework of the chiral perturbation theory (CHPT) by Bijmans, Ecker, and Gasser [1]. The lowest order approximation $O(p^2)$ of CHPT gives the $K_{l3\gamma}$ amplitude independent of free parameters except for the Fermi coupling $G_F V_{us}$ for the us currents. The $O(p^4)$ corrections to the $K_{l3\gamma}$ amplitude, calculated in [1], include three general types of contributions: anomaly, local contributions due to \mathcal{L}_4 and loop amplitudes.

The aim of this paper is to analyze the possibility of the experimental isolation of these contributions. We will follow the notation of [1]. To parameterize the local \mathcal{L}_4 and anomaly contributions the dimensionless constants

$$l_9 = \frac{4M_K^2}{\sqrt{2}F^2} \left(L_9^r(\mu) - \frac{1}{256\pi^2} \ln \frac{M_\pi M_K^2 M_\eta}{\mu^4} \right) = 0.64 \pm 0.01,$$

$$l_{10} = \frac{4M_K^2}{\sqrt{2}F^2} (L_9^r(\mu) + L_{10}^r(\mu)) = 0.11 \pm 0.02, \quad (1)$$

$$l_A = \frac{\sqrt{2}M_K^2}{16\pi^2 F^2} = 0.25$$

will be used. Here, $F = 93.2$ MeV is the pion decay constant, $M_{\pi, K, \eta}$ are the meson masses, and $L_{9, 10}^r$ are the renormalized low-energy couplings of the CHPT \mathcal{L}_4 lagrangian. The above constants (1) are independent of the renormalization scale μ . The numerical estimates of l_9 and l_{10} come from the experimental data on the pion

charge radius $\langle r_\pi^2 \rangle \sim L_9$ and the ratio of the axial to vector form factors $\gamma = 32\pi^2(L_9 + L_{10})$ in the $\pi \rightarrow e\nu\gamma$ decay, respectively. We define l_{10} to be proportional to the sum $L_9 + L_{10}$, not to the L_{10} , because (i) just this sum is measured in $\pi \rightarrow e\nu\gamma$ decay and (ii) the parameters l_9 and l_{10} are less correlated in $K_{l3\gamma}$ decay than L_9 and L_{10} . As will be shown below, the loop amplitude contribution is small and virtually cannot be detected in $K_{l3\gamma}$ decay.

The most general (neglecting the high-order electroweak corrections) matrix element of the $K_{l3\gamma}^+$ decay is given by the equation [1]:

$$T = \frac{G_F}{\sqrt{2}} e V_{us}^* \epsilon^\mu(q)^* \times \left\{ (V_{\mu\nu} - A_{\mu\nu}) \bar{u}(p_\nu) \gamma^\nu (1 - \gamma_5) v(p_l) \right. \\ \left. + \frac{E_\nu^+}{2p_l q} \bar{u}(p_\nu) \gamma^\nu (1 - \gamma_5) (m_l - p_l' - \not{q}) \gamma_\mu v(p_l) \right\}, \quad (2)$$

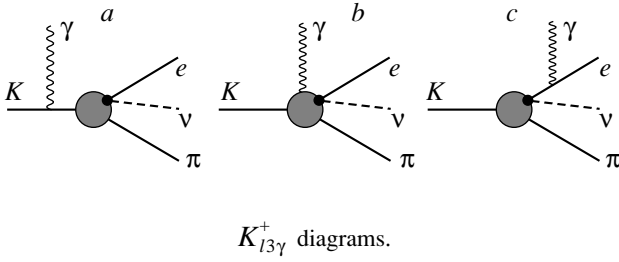
where ϵ^μ denotes the polarization vector of the photon, $V_{\mu\nu}$ and $A_{\mu\nu}$ are the hadronic tensors, corresponding to the vector and axial-vector amplitudes, and

$$F_\nu^+(t) = \frac{1}{\sqrt{2}} \{ (p + p')_\nu f_+(t) + (p - p')_\nu f_-(t) \}, \quad (3)$$

$$t = (p - p')^2$$

is the K_{l3}^+ matrix element.

The second part of equation (2) corresponds to the lepton Bremsstrahlung diagram of figure *c*, while the first one includes both the Bremsstrahlung of the K^+ (*a*) and the figure Structure Dependent terms (*b*).



2. INNER BREMSSTRAHLUNG

Due to Low's theorem [2] the part of the $K_{l3\gamma}$ amplitude, containing the terms of order $O(q^{-1}, q^0)$ (Inner Bremsstrahlung) (IB) is unambiguously defined by the K_{l3} amplitude, i.e., by the form factors $f_{\pm}(t)$. Obviously, K_{l3} decay is much more suitable for the experimental study of these form factors than $K_{l3\gamma}$ because (i) K_{l3} is two orders of magnitude more intensive process than $K_{l3\gamma}$, (ii) additional γ in $K_{l3\gamma}$ cause the problem of mismatching of the "radiative" photon and the photon from π^0 decay, (iii) K_{l3} is less sensitive to the background associated with faked photons, (iv) some backgrounds, substantial for $K_{l3\gamma}$ (e.g. $K^+ \rightarrow \pi^+\pi^0\pi^0$ decay) are negligible for K_{l3} . For this reason we will consider the form factors $f_{\pm}(t)$ and, consequently, the IB amplitude as apriori known, which are not to be the subject of the $K_{l3\gamma}$ experiment.

Nonetheless, it may be useful to get understanding about the CHPT contributions to the form factors $f_{\pm}(t)$. To the lowest order $O(p^2)$ approximation $f_+ = 1, f_- = 0$. The $O(p^4)$ approximation in CHPT gives [1]:

$$\begin{aligned} f_+(t) &= 1 + l_9 t / M_K^2 + (-0.023 - 0.004 t / m_\pi^2)_{\text{loop}}, \\ f_-(t) &= (F_K / F_\pi - 1) - l_9 (1 - m_\pi^2 / M_K^2) \\ &\quad + (0.029 + 0.003 t / m_\pi^2)_{\text{loop}}. \end{aligned} \quad (4)$$

The $F_K / F_\pi - 1 = 0.22$ is the parametrization of the contribution of the counterterm L_5 . The subscript loop is used to specify the contributions of the loop amplitudes. The quadratic on t terms are negligible, so the form factors satisfy the traditional parametrization $f_{\pm}(t) = f_{\pm}(0)(1 + \lambda_{\pm} t / m_\pi^2)$.

Without loop contributions $f_+(t) = 1 + 0.034 t / m_\pi^2$, while in full $O(p^4)$ amplitude $f_+(t) = 0.977(1 + 0.031 t / m_\pi^2)$. The correction to λ_+ due to the loop contributions noticeably improves the consistency between the CHPT prediction and the experimental value $\lambda_+^{\text{exp}} = 0.029 \pm 0.002$ [3], but this result is on the level of experimental errors and possible high order correction. More important is the loop amplitude correction to $f_+(0)$. It gives $\sim 5\%$ correction to the total decay probability and it is very significant for the determination of the element V_{us} of Cabibbo–Kobayashi–Maskawa mixing matrix [4].

The experiment is sensitive to $f_-(t)$ only in combination $f_0(t) = f_+(t) + f_-(t)t / (m_K^2 - m_\pi^2) = f_0(0)[1 + \lambda_0 t / m_\pi^2]$. The l_9 does not contribute to $f_0(t)$, and prior to the loop corrections $f_0(t) = 1 + 0.0177 t / m_\pi^2$. Taking into account the loop contributions one can get $f_0(t) = 0.996 f_+(0)(1 + 0.0189 t / m_\pi^2)$. The significant spread in the experimental measurements of λ_0 [3] does not allow to examine the loop corrections to this parameter.

The IB amplitude corresponds to the diagrams presented in Figs. *a* and *c*. Following the directions of the Low's theorem one can get the IB part of the vector amplitude $V_{\mu\nu}$:

$$\begin{aligned} V_{\mu\nu}^{\text{IB}} &= F_v(t) \frac{p_\mu}{pq} + C_2(t) \left(\frac{p_\mu q_\nu}{pq} - g_{\mu\nu} \right) \\ &\quad + 2qW \left(\frac{p_\mu}{pq} - \frac{W_\mu}{qW} \right) (C_1'(t)p'_\nu + C_2'(t)W_\nu), \end{aligned} \quad (5)$$

where $t = W_q^2 = (W + q)^2$, $W^\mu = (p_l + p_\nu)^\mu$, $C_1(t) = \sqrt{2}f_+(t)$, $C_2(t) = (f_+(t) + f_-(t))/\sqrt{2}$ and C_i' are the derivatives of $C_i(t)$. The axial amplitude does not contribute to IB.

3. STRUCTURE DEPENDENT RADIATION

The hadronic tensors may be decomposed on the Inner Bremsstrahlung and the Structure Dependent (SD) parts:

$$\begin{aligned} V_{\mu\nu} &= V_{\mu\nu}^{\text{IB}} + V_1^{\text{SD}} (p'_\mu q_\nu - p'_\nu q_\mu g_{\mu\nu}) \\ &\quad + V_2^{\text{SD}} (W_\mu q_\nu - qW g_{\mu\nu}) \\ &\quad + V_3^{\text{SD}} (qW p'_\mu W_\nu - p'_\nu qW_\mu W_\nu) \\ &\quad + V_4^{\text{SD}} (qW p'_\mu p'_\nu - p'_\nu qW_\mu p'_\nu), \\ A_{\mu\nu} &= i\epsilon_{\mu\nu\rho\sigma} (A_1^{\text{SD}} p^\rho q^\sigma + A_2^{\text{SD}} q^\rho W^\sigma) \\ &\quad + i\epsilon_{\mu\lambda\rho\sigma} p'^\lambda q^\rho W^\sigma (A_3^{\text{SD}} W_\nu + A_4^{\text{SD}} p'_\nu). \end{aligned} \quad (6)$$

The four invariant vector amplitudes V_i^{SD} and four axial amplitudes A_i^{SD} define the most general amplitude of the SD radiation in $K_{l3\gamma}$ decay. According to the CHPT calculations [1]:

$$\begin{aligned} V_1^{\text{SD}} &= I_2, \\ V_2^{\text{SD}} &= -(I_1 + I_2 p'q) / qW, \\ V_3^{\text{SD}} &= (I_3 - f_2(W^2)) / qW, \\ V_4^{\text{SD}} &= 0, \end{aligned} \quad (8)$$

where $I_{1,2,3}$ and $f_2(W^2)$ are defined in equations (5.21) and (5.18) of [1], respectively.

Separating the local and loop contributions, we found:

$$V_{\mu\nu}^{\text{SD}} = \frac{2l_9}{M_K^2}(p'_\mu q_\nu - p'_\nu q_\mu g_{\mu\nu}) + \frac{l_{10}}{M_K^2}(W_\mu q_\nu - q W_\mu g_{\mu\nu}) + l_{\text{loop}} V_{\mu\nu}^{\text{loop}}, \quad (9)$$

where $V_{\mu\nu}^{\text{loop}}$ is the loop part of equation (8), and $l_{\text{loop}} = 1$ is used to parameterize it. The axial amplitude is given by the chiral anomaly [1]:

$$A_{\mu\nu} = i \frac{l_A}{M_K^2} \times \left\{ \epsilon_{\mu\nu\rho\sigma} q^\rho (4p' + W)^\sigma + \frac{4}{W^2 - M_K^2} \epsilon_{\mu\lambda\rho\sigma} W_\nu p'^\lambda q^\rho W^\sigma \right\}. \quad (10)$$

4. NUMERICAL ESTIMATES

The calculations of the $K_{l3\gamma}$ decay probabilities have been performed using the FORTRAN code for calculation of the square of the $K_{l3\gamma}$ matrix element, obtained from G. Ecker.

The decay probabilities for the SD terms only are very small compared with the IB one (Table 1). So, the only way for the experimental detection of the SD contributions to the decay rate is the observation of their in-

terference with IB term. Suggesting the study of the deviation of the experimental decay rate from the theoretical prediction for the IB radiation, it is necessary to keep in mind possible high order corrections to the IB term as well as systematic errors of the measurements. So, only the regions of the phase space with relatively large contribution of the SD term are actually suitable for analysis. For this reason we estimate the contribution of the SD terms (mainly due to the interference with IB) for two phase space regions $\Phi_{0.05}$ and $\Phi_{0.2}$. These regions Φ_η are defined (for each considered component of the SD radiation independently) by the condition:

$$|\mathcal{M}_{\text{IB+SD}}^2 - \mathcal{M}_{\text{IB}}^2| / \mathcal{M}_{\text{IB}}^2 > \eta, \quad (11)$$

where \mathcal{M}^2 is the square of the matrix element.

The results of calculation of the IB Branching Ratios (Br^{IB}) and corrections due to SD term (ΔBr) for the selected regions of the phase space are shown in Table 1. These values allow to estimate the experimental statistics necessary to detect the SD radiation. To get the idea about the backgrounds tolerable to perform the isolation of the SD terms, the effective (assuming constant matrix element) Branching Ratios for the total phase space (Br^{eff}), which give the same contribution to selected phase space region as ΔBr , are also displayed in Table 1. Uncontrolled background at the level of branching ratio Br^{eff} will make impossible the experimental measurement of the Structure Dependent contributions to the $K_{l3\gamma}$ decay rate.

The $K \rightarrow l\nu\pi^0\pi^0$ decay ($\text{Br} \sim 10^{-5}$) is very important “natural” background for the $K_{l3\gamma}$ decays. If the ef-

Table 1. Contributions of the l_9 , l_{10} , anomaly and loop amplitudes to the $K_{l3\gamma}^+$ Branching ratio (Br^{SD} are decay probabilities for SD term alone, Br^{IB} are probabilities of IB terms in the considered region of the phase space, ΔBr are the corrections to the IB probabilities due to the SD terms, and Br^{eff} are the effective (assuming constant matrix element) Branching ratios which gave the same contribution to the selected region of phase space as ΔBr)

$K_{e3\gamma}^+$	Br^{SD}	$\Phi_{0.05}$			$\Phi_{0.2}$		
		ΔBr	Br^{IB}	Br^{eff}	ΔBr	Br^{IB}	Br^{eff}
l_9	5.5×10^{-8}	-8.0×10^{-7}	9.8×10^{-6}	1.9×10^{-6}	-2.1×10^{-8}	9.3×10^{-8}	5.1×10^{-7}
l_{10}	2.6×10^{-10}	-4.8×10^{-10}	7.7×10^{-9}	4.5×10^{-8}	-2.1×10^{-14}	1.0×10^{-13}	3.7×10^{-9}
Anomaly	4.3×10^{-8}	-4.4×10^{-7}	5.7×10^{-6}	1.4×10^{-5}	-1.5×10^{-8}	7.8×10^{-8}	4.3×10^{-7}
Loops	4.5×10^{-10}	1.8×10^{-11}	2.6×10^{-10}	1.3×10^{-8}	4.4×10^{-13}	1.7×10^{-12}	9.4×10^{-9}
Total	9.3×10^{-8}	-1.9×10^{-6}	1.9×10^{-5}	3.8×10^{-6}	-3.3×10^{-7}	1.2×10^{-6}	2.3×10^{-6}
$K_{\mu3\gamma}^+$	Br^{SD}	$\Phi_{0.05}$			$\Phi_{0.2}$		
		ΔBr	Br^{IB}	Br^{eff}	ΔBr	Br^{IB}	Br^{eff}
l_9	1.3×10^{-8}	-1.6×10^{-7}	2.0×10^{-6}	4.3×10^{-7}	-1.7×10^{-9}	1.1×10^{-8}	1.1×10^{-7}
l_{10}	1.1×10^{-10}	-5.5×10^{-11}	1.1×10^{-9}	1.2×10^{-8}	—	—	—
Anomaly	1.2×10^{-8}	-1.5×10^{-7}	1.8×10^{-6}	4.0×10^{-7}	-6.7×10^{-9}	2.9×10^{-8}	1.7×10^{-7}
Loops	9.1×10^{-11}	-1.8×10^{-13}	3.1×10^{-12}	7.4×10^{-9}	—	—	—
Total	2.5×10^{-8}	-4.3×10^{-7}	4.0×10^{-6}	8.1×10^{-7}	-8.7×10^{-8}	3.3×10^{-7}	6.0×10^{-7}

Table 2. Possible BNL E865 experiment sensitivity (1 sigma) to the Structure Dependent $K_{l3\gamma}$ radiation (The expected mean values of factors l are shown in parantheses)

Decay	$K_{e3\gamma}$		$K_{\mu3\gamma}$	
Eff. number of K decays	10^8		10^9	
Background level	–	10^{-6}	–	10^{-5}
δl_{tot} (1.00)	0.19	0.20	0.12	0.19
δl_9 (0.64)	0.21	0.22	0.15	0.29
δl_{10} (0.11)	0.62	0.71	0.39	0.99
δl_A (0.25)	0.10	0.11	0.06	0.12
δl_{loop} (1.00)	3.65	3.78	2.76	5.18
δl_9^{corr}	0.53	0.60	0.30	0.94
δl_A^{corr}	0.26	0.29	0.11	0.39

fective mass of neutrino and missed photon is small, such background process is kinematically indistinguishable from the $K_{l3\gamma}$. So, the experimental measurements of the l_{10} and loop contributions to the SD amplitude seem to be unlikely, at least, without specially designed experimental setup, including high efficiency veto system for photons. Definitely, there is certain incompatibility of such veto system with the requirement of the high intensity kaon flux, necessary to perform the high statistic experiment.

For the $K_{\mu3\gamma}$ decay there is another important background related to the $K \rightarrow \pi\pi^0\pi^0$ decay ($\text{Br} = 0.017$) followed by the $\pi_{\mu2}$ decay within the experimental setup. This background can make impossible even the measurement of the total SD contribution to the $K_{\mu3\gamma}$ decay.

The $K_{l3\gamma}^+$ decays may be studied by product in the BNL E865 experiment (search for lepton number violation decay $K^+ \rightarrow \pi^+\mu^+e^-$). The effective number of kaon decays (recalculated for the detector 100% acceptance and efficiency) is being expected up to $\sim 10^8$ for $K_{e3\gamma}$ decay and $\sim 10^9$ for the $K_{\mu3\gamma}$. The difference in the effective numbers of the decays is mainly due to the trigger conditions, required by the $K \rightarrow \pi\mu e$ process detection. Using a normalization factor $l_{\text{tot}} = 1$ for the total SD amplitude, we can evaluate the possible statistical sensitivity δl_{tot} (one standard deviation) to the Structure Dependent radiation in $K_{l3\gamma}$ decay. The same estimates have been done for the each component of the SD amplitude (assuming all other components to be fixed) and for the case of simultaneous measurement of the contributions due to L_9 and anomaly (δl_9^{corr} and δl_A^{corr}). The results are shown in Table 2 and should be compared with suggested values (1). Only the phase space region, where the correction to the decay rate due to total SD term exceeds 0.05, was taken into account. The sensitivity of the experiment to the SD parameters was also estimated for the case of the well normalized

background presented. The level of the backgrounds 10^{-6} and 10^{-5} was chosen for the $K_{e3\gamma}$ and $K_{\mu3\gamma}$ decays, respectively, in a rough accordance with the above discussion of the expected backgrounds. The systematic uncertainties in the backgrounds were not taken into account. So, the obtained results may be considered only as an upper (optimistic) limit for the accuracy of the experimental study of the Structure Dependent radiation in the $K_{l3\gamma}$ decay for the suggested effective numbers of the decays. Obviously, the obtained accuracy of the measurements depends on the number of events like $\sim 1/\sqrt{N}$.

The quoted number [1] of the annual kaon flux at DAΦNE allows to improve the statistical accuracy of measurements by factor 10. But taking into account the real acceptance of the experimental setup, the detection efficiency, the trigger selection, and possible uncertainties in the background evaluation one could not expect any significant improvement in the results of the possible analysis of the Structure Dependent radiation in the $K_{l3\gamma}$ decay.

5. CONCLUSION

According to Table 2 it is possible to isolate the SD contribution to the $K_{l3\gamma}$ decay as a correction ($\sim 10\%$) to the IB decay rate. Probably, it will be also possible to check separately the contributions due to the L_9 coupling and anomaly at the confidence level of 2–3 standard deviations. It is also expected that experiment will not allow to detect reliably the l_{10} and the loop amplitude contribution to SD radiation. A simultaneous study of the L_9 counterterm and anomaly contributions is also doubtful. All these estimates are based on the CHPT predictions [1] for radiative K_{l3} decay.

Our conclusion about the significance of the loop diagram contribution to the $K_{l3\gamma}$ differs from the conclusion of authors of [1]. The almost 10% change of the

total decay rate provided by loop corrections [1] is mainly predefined by the correction to the K_{l3} form factor $f^+(t)$. As was discussed above, this form factor should be measured in K_{l3} decay rather than in $K_{l3\gamma}$. Also, we can point out that if $K_{l3\gamma}$ decay rate will be experimentally normalized using K_{l3} decay, the mentioned effect of the loop diagrams will be significantly canceled.

The loop diagram effects may be found by measuring the form factor slope λ^+ and total decay rate ($f^+(0)$). The first way requires improvement of the experimental accuracy as well as a study of high order corrections. To check the loop contribution to the total decay rate

we need at least an independent determination of the element V_{us} of Cabibbo–Kobayashi–Maskawa matrix.

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ЧТО МОЖНО ИЗУЧАТЬ В РАДИАЦИОННОМ K_{l3} -РАСПАДЕ?

А. А. Поблагуев

Обсуждается возможность проверки киральной теории возмущений при экспериментальном изучении структурного излучения в распаде $K^+ \rightarrow l^+ \nu \pi^0 \gamma$. Показано, что эксперимент может быть чувствительным только к вкладам, соответствующим контрчлену L_9 и аксиальной аномалии. Однако одновременное независимое изучение этих вкладов представляется маловероятным.